

EXTENDS *Sequences, Naturals*

CONSTANTS *Sequence*

ASSUME $Sequence \in Seq(Nat)$

$IsIncreasingSubsequence(indices) \triangleq$
 $\wedge Len(indices) \leq Len(Sequence)$
 $\wedge \forall i \in 1 .. Len(indices) :$
 $\quad indices[i] \leq Len(Sequence)$
 $\wedge \vee Len(indices) \leq 1$
 $\quad \vee \wedge Len(indices) > 1$
 $\quad \wedge \forall i \in 1 .. Len(indices) - 1 :$
 $\quad \quad indices[i] < indices[i + 1]$
 $\quad \wedge \forall i \in 1 .. Len(indices) - 1 :$
 $\quad \quad Sequence[indices[i]] < Sequence[indices[i + 1]]$

$IsSolution(candidate) \triangleq$
 LET $length \triangleq Len(candidate)$
 $subsequences \triangleq UNION \{[1 .. n \rightarrow 1 .. Len(Sequence)] : n \in 0 .. Len(Sequence)\}$
 IN $\wedge IsIncreasingSubsequence(candidate)$
 $\wedge \forall subsequence \in subsequences :$
 $\quad \vee \neg IsIncreasingSubsequence(subsequence)$
 $\quad \vee \wedge IsIncreasingSubsequence(subsequence)$
 $\quad \quad \wedge Len(subsequence) \leq length$

VARIABLES *candidates, solutions*

$Init \triangleq$
 $\wedge candidates = \{\langle n \rangle : n \in 1 .. Len(Sequence)\}$
 $\wedge solutions = \{\}$

$Extend \triangleq \exists candidate \in candidates :$
 LET $start \triangleq candidate[Len(candidate)]$
 $highest \triangleq Sequence[start]$
 $options \triangleq \{n \in (start + 1) .. Len(Sequence) : Sequence[n] > highest\}$
 $extensions \triangleq \{Append(candidate, option) : option \in options\}$
 IN IF $extensions = \{\}$
 THEN
 LET $updated \triangleq solutions \cup \{candidate\}$
 $lengths \triangleq \{Len(solution) : solution \in updated\}$
 $filtered \triangleq \{solution \in updated : (\forall length \in lengths : Len(solution) \geq length)\}$
 IN $\wedge candidates' = candidates \setminus \{candidate\}$
 $\wedge solutions' = filtered$
 ELSE
 $\wedge candidates' = (candidates \setminus \{candidate\}) \cup extensions$
 $\wedge UNCHANGED solutions$

$$\begin{aligned} \text{Spec} &\triangleq \text{Init} \wedge \Box[\text{Extend}]_{(candidates, solutions)} \\ \text{Invariant} &\triangleq candidates = \{\} \Rightarrow \forall solution \in solutions : \\ &\quad \text{IsSolution}(solution) \\ \text{Termination} &\triangleq \Diamond(candidates = \{\}) \\ \text{NoSolutions} &\triangleq solutions = \{\} \end{aligned}$$